What Makes an Asymptote?

First, Sort & Compare...

Then...
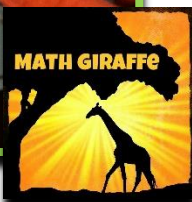What makes a horizontal asymptote?
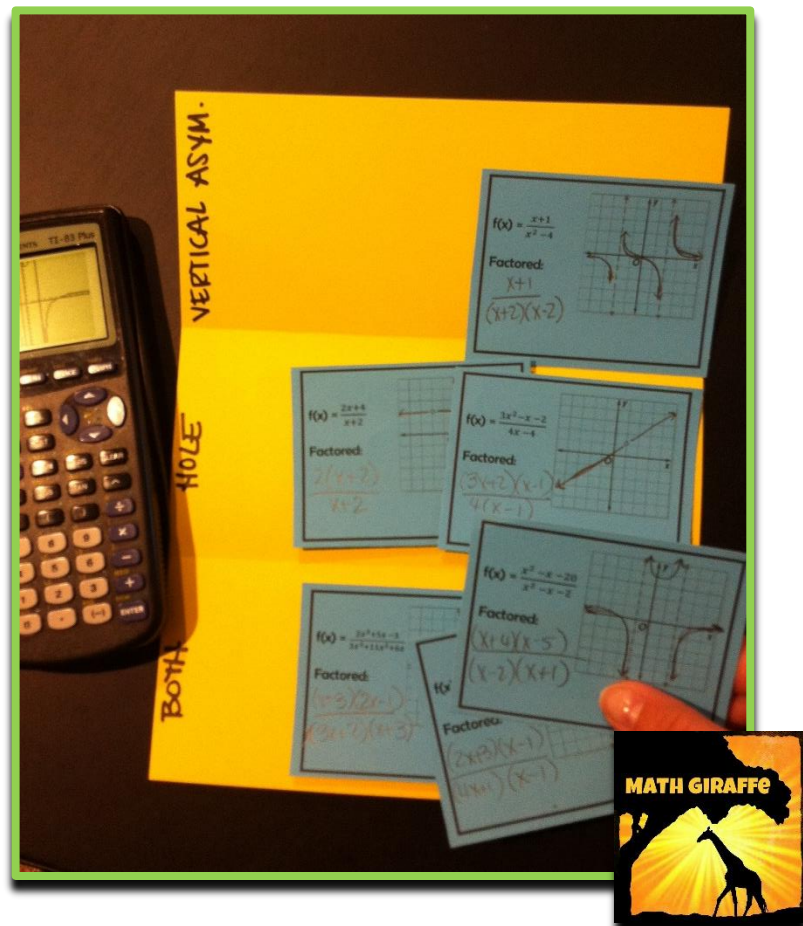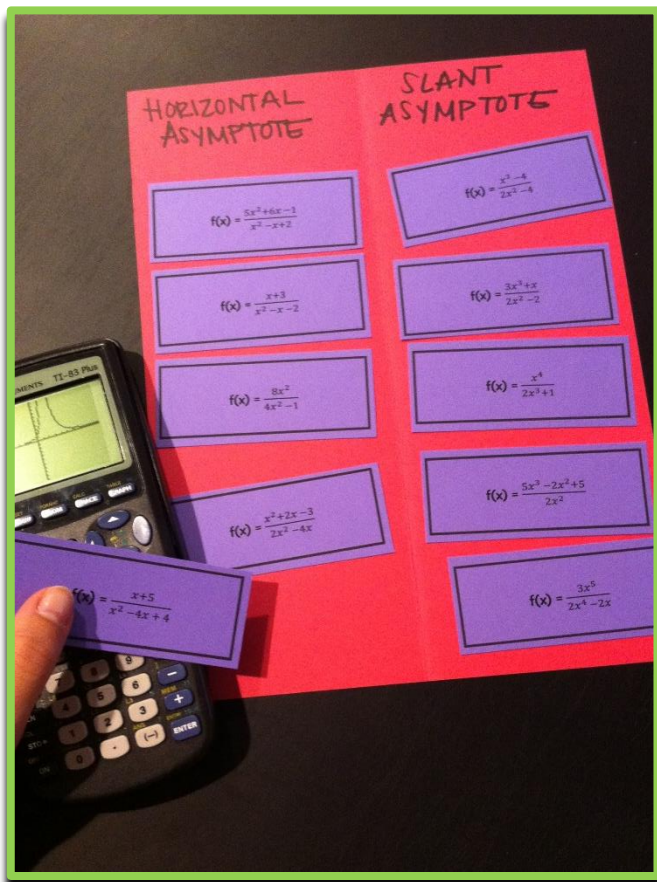What makes a slant asymptote?
What makes a vertical asymptote?
What makes a hole?

Next...
How can we find where each will be?

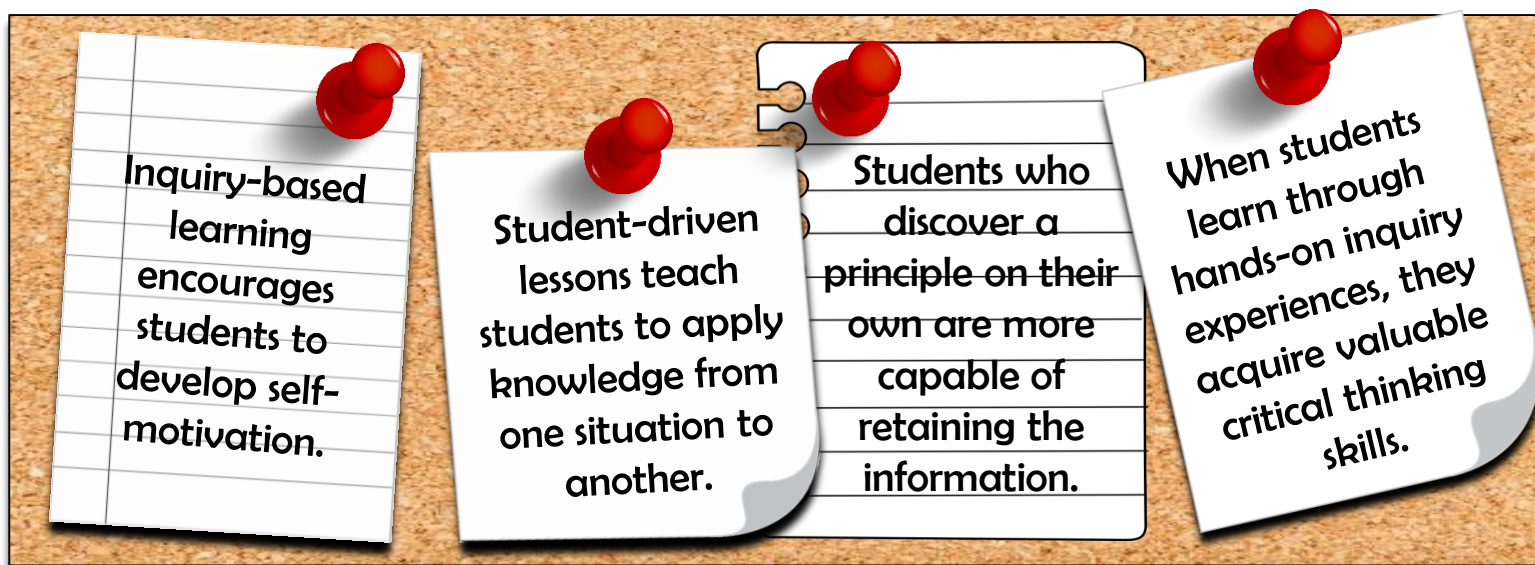# Guided Inquiry with Rational Functions

About Inquiry Based Learning

MATH GIRAFFE

When you use an inquiry-based approach in your classroom, students discover mathematical properties for themselves. Instead of presenting students with a theorem, formula, or rule, you guide them through an investigation. Inquiry learning is often done in a hands-on way.

When students develop a rule for themselves, they understand the concept behind it more deeply. Also, since they have produced the rule or formula themselves, they do not need to memorize it. They know how it was developed and can reproduce it at any time. This deeper understanding of mathematical properties helps them to approach new problems in the future.

In my inquiry lessons, I like to have students write patterns and observations in complete sentences, then develop a rule for the property they have observed. This helps them to internalize what is happening. Then, I lead them into writing a mathematical formula or rule to represent their words. The students learn to look for connections. They also develop valuable skills such as the ability to self-direct and the ability to clearly express their discoveries in both words and mathematical language.

If you are interested in learning more about inquiry-based learning in the math classroom, head over to mathgiraffe.com and browse my blog. While you're there, you can also sign up for my email list.

Inquiry-based learning encourages students to develop self-motivation.

Student-driven lessons teach students to apply knowledge from one situation to another.

Students who discover a principle on their own are more capable of retaining the information.

When students learn through hands-on inquiry experiences, they acquire valuable critical thinking skills.

Note to Teacher:

This lesson is an introduction to graphing rational functions. Use it before students are familiar with asymptotes.

Students will need graphing calculators.

Start by defining asymptotes and show a few examples. Then, give students the Cards for Investigation 1. It works best if they cut them apart and sort them, so they can easily compare characteristics. Students work on their own to discover what parts of the rational function cause the vertical asymptotes, and what causes the holes.

Then, students can complete Investigation 2. They use their graphing calculator to determine which functions have a horizontal asymptote, and which have a slant asymptote. They figure out what features within the function cause each type.

Try to avoid giving too many hints, but you can try offering suggestions to students/pairs who are really stuck. Ask them which terms will become more "powerful" as x approaches infinity or negative infinity. Have them look more closely at the factors or degrees as needed.

Come back together as a whole class to share discoveries and clear up any misconceptions. This is the time to have students take notes.

Rational Function Cards: Investigation 1

Holes and vertical asymptotes are shown for you. Use your graphing calculator to complete a sketch of each graph. Show each rational function in factored form. What causes a vertical asymptote? What causes a hole in the graph?
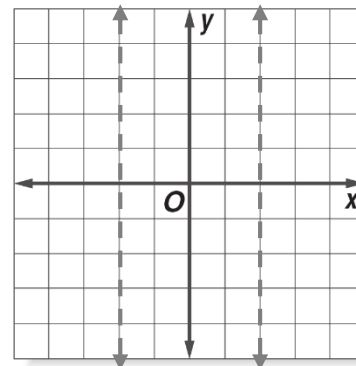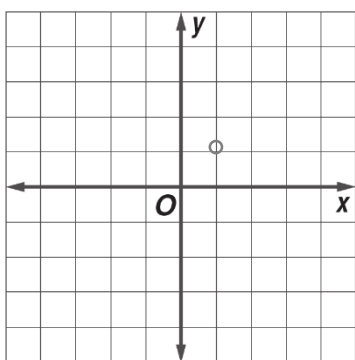
$f(x) = \dfrac{2x^2 + x - 3}{4x^2 - 3x - 1}$
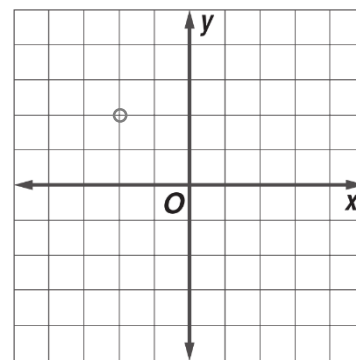
Factored:
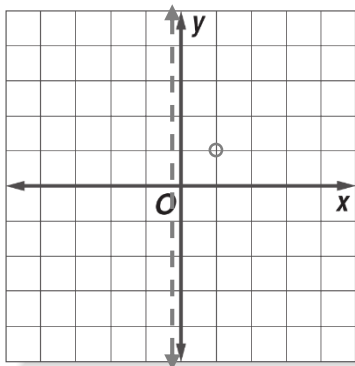
$f(x) = \dfrac{x + 1}{x^2 - 4}$

Factored:

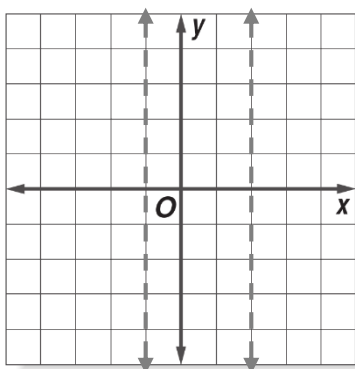$f(x) = \dfrac{3x^2 - x - 2}{4x - 4}$

Factored:

$f(x) = \dfrac{2x + 4}{x + 2}$

Factored:

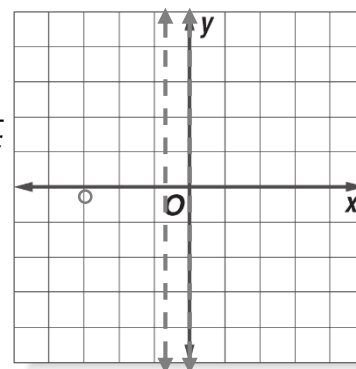$f(x) = \dfrac{x^2 - x - 20}{x^2 - x - 2}$

Factored:

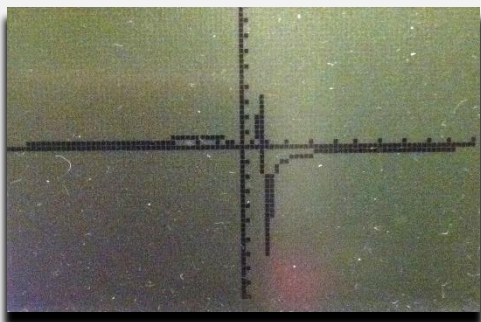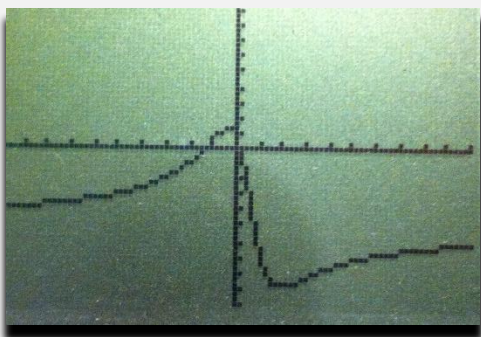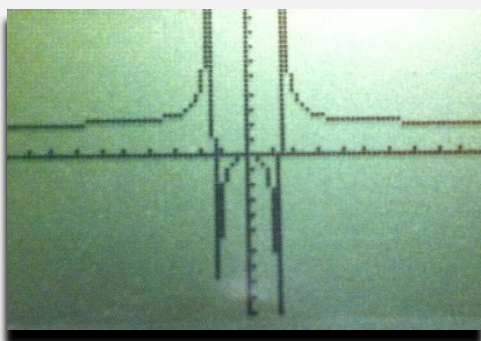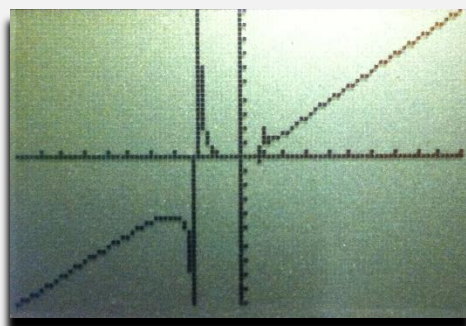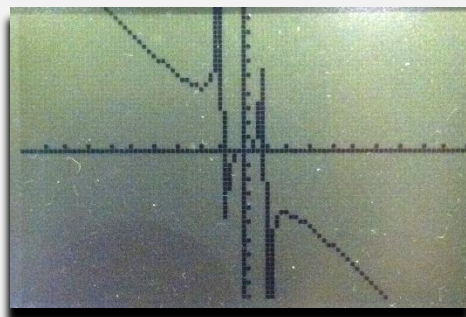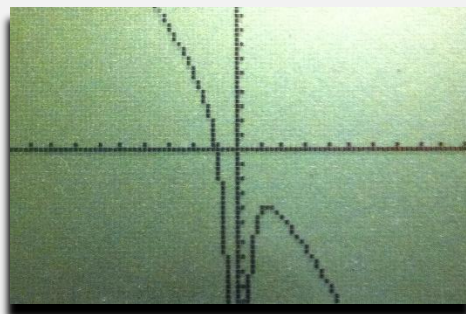$f(x) = \dfrac{2x^2 + 5x - 3}{3x^3 + 11x^2 + 6x}$

Factored:

When looking at the equation for a rational function, how can you tell WHERE the vertical asymptote(s) will be? How can you tell where the holes will be?

Reference Sheet:
Samples of Horizontal and Slant Asymptotes

Horizontal Asymptotes

Slant Asymptotes

Rational Function Cards
Investigation 2

Use your graphing calculator to graph each rational function. Use the reference sheet for samples of horizontal and slant asymptotes. Determine which type of asymptote is present in each rational function. What causes a horizontal asymptote? What causes a slant asymptote?

f(x) = $\dfrac{x^2+2x-3}{2x^2-4x}$

f(x) = $\dfrac{x^3-4}{2x^2-4}$

f(x) = $\dfrac{x+5}{x^2-4x+4}$

f(x) = $\dfrac{8x^2}{4x^2-1}$

f(x) = $\dfrac{3x^5}{2x^4-2x}$

f(x) = $\dfrac{x+3}{x^2-x-2}$

f(x) = $\dfrac{x^4}{2x^3+1}$

f(x) = $\dfrac{5x^3-2x^2+5}{2x^2}$

f(x) = $\dfrac{3x^3+x}{2x^2-2}$

f(x) = $\dfrac{5x^2+6x-1}{x^2-x+2}$

Can you determine how to tell WHERE a horizontal asymptote will be? How can you find the equation of the line for a slant asymptote?

Additional Resources:

These items would also be great for your Algebra classroom this year!

Click the images to learn more about these items.

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Identifying Asymptotes Activity

Cut, factor, analyze degrees, match types of asymptotes & holes, and then write your own rational functions!

GridWords: Factoring

FULL SET

Simplifying Rational Expressions

GOLF GAME

Driver Player A

Iron

Putter Player B

Player A

Hole 1

Score: (add strokes)

3rd: Putter

Draw/putt until correct. Record number of cards as strokes.

2nd: Iron

If incorrect, record 2 strokes (in the sand)

1st: Driver

If incorrect, record 2 strokes (in the rough)

Driver   Player A

$$\frac{6x^2 + 9x + 2}{4x^2 + 4x}$$